ENERGY PRINCIPLE

$$\text{The term enthalpy} = h = \left(u + \frac{p}{\rho}\right)$$

$$\dot{Q} - \dot{W}_s = \int_{A2} \left(\frac{p_2}{\rho} + u_2 + \frac{V_2^{\ 2}}{2} + gz_2\right)\rho V_2 dA_1 - \int_{A1} \left(\frac{p_1}{\rho} + u_1 + \frac{V_1^{\ 2}}{2} + gz_1\right)\rho V_1 dA_1$$

$$\dot{Q} - \dot{W}_s + \int_{A1}\left(\frac{p_1}{\rho} + gz_1 + u\right)\rho V_1 dA_1 + \int_{A1} \frac{\rho V_1^{\ 3}}{2} dA_1 = \int_{A2} \left(\frac{p_2}{\rho} + gz_2 + u_2\right)\rho V_2 dA_1 + \int_{A2} \frac{\rho V_2^{\ 3}}{2} dA_2$$

It is common to express the term $\int \frac{\rho V^3}{2} dA = \alpha\left(\frac{\overline{V}^3}{2}\right) A$

Substituting for $\int \frac{\rho V^3}{2} dA = \alpha\left(\frac{\overline{V}^3}{2}\right) A$ **in Eqn. above and dividing through by** $\dot{m}$

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\overline{V}_1^{\ 2}}{2}\right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\overline{V}_2^{\ 2}}{2}\right)$$

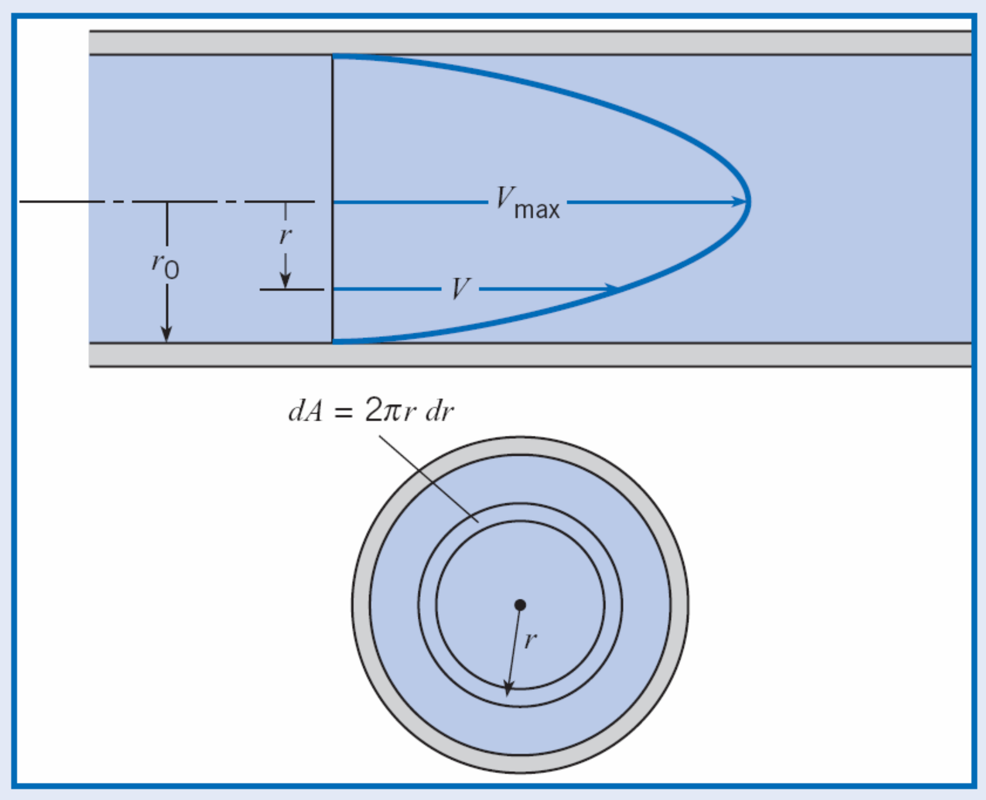
Example (7.2)

The velocity distribution for laminar flow in a pipe is given by the equation

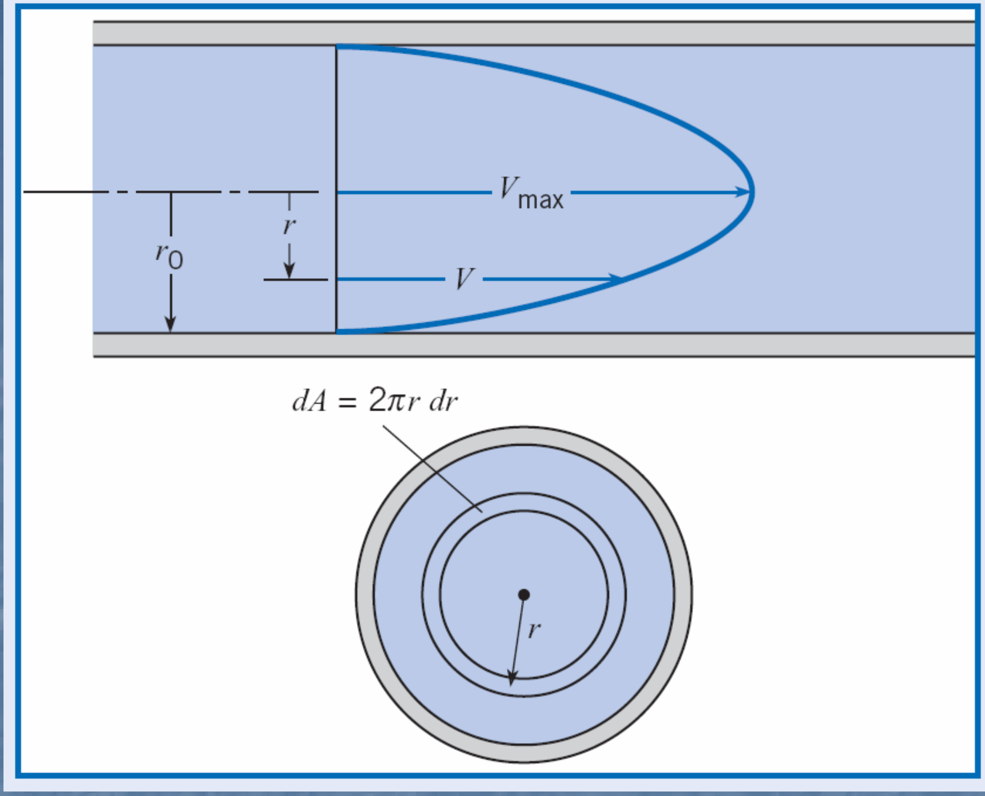
$$V = V_{max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

Here r_0 is the radius of the pipe and r is the radial distance from the center. Determine $\overline{V}$ in terms of V_{max} and evaluate the kinetic-energy correction factor α.

Find: $\overline{V}$ *in terms of* V_{max}, α



Given: $V = V_{\max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$



Solution

$$Q = A\overline{V} = \int_0^{r_0} V_{\max}\left[1-\left(\frac{r}{r_0}\right)^2\right](2\pi r)dr$$

$$\overline{V} = \frac{-\pi}{A} r_0^2 V_{\max} \frac{(1-r^2/r_0^2)^2}{2}\Bigg|_0^{r_0} = \frac{1}{2}V_{\max}$$

$$\overline{V} = \frac{1}{2}V_{\max}$$

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\overline{V}}\right)^3 dA = \left(\frac{1}{\pi r_0^2}\right)\int_0^{r_0}\left[\frac{V_{\max}^3\left(1-\left(\frac{r}{r_0}\right)^2\right)^3}{\left(\frac{1}{2}V_{\max}\right)^3}\right](2\pi r)dr = 2$$

ENERGY PRINCIPLE

Pump – Turbine Combination

When a compressor and a turbine are connected together, the resultant shaft work $\dot{W}_S$ is equal to

$$\dot{W}_S = \dot{W}_T - \dot{W}_p$$

Substituting for the above expression in Eqn. below

$$\frac{1}{\dot{m}}(\dot{Q} - \dot{W}_s) + \left(\frac{p_1}{\rho} + gz_1 + u_1 + \alpha_1 \frac{\overline{V}_1^{\,2}}{2}\right) = \left(\frac{p_2}{\rho} + gz_2 + u_2 + \alpha_2 \frac{\overline{V}_2^{\,2}}{2}\right)$$

and divide by (g), we have

$$\frac{\dot{Q}}{\dot{m}g} - \left(\frac{\dot{W}_T - \dot{W}_P}{g}\right) + \left(\frac{p_1}{\rho g} + z_1 + \frac{u_1}{g} + \alpha_1 \frac{\overline{V}_1^{\,2}}{2g}\right) = \left(\frac{p_2}{\rho g} + gz_2 + \frac{u_2}{g} + \alpha_2 \frac{\overline{V}_2^{\,2}}{2g}\right)$$

ENERGY PRINCIPLE

Rearranging, we have

$$\left(\frac{\dot{W}_p}{\dot{m}g}\right)+\left(\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V_1}^2}{2g}\right)=\left(\frac{\dot{W}_T}{\dot{m}g}\right)+\left(\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V_2}^2}{2g}\right)+\left(\frac{u_2-u_1}{g}\right)-\left(\frac{\dot{Q}}{\dot{m}g}\right)$$

$$\left(h_P+\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V_1}^2}{2g}\right)_{Mech.part}=\left(h_T+\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V_2}^2}{2g}\right)_{Mech.part}+\left(\frac{u_2-u_1}{g}-\frac{\dot{Q}}{\dot{m}g}\right)_{thermal\ part}$$

$$h_P=\frac{\dot{W}_C}{\dot{m}g}=Pump\ Head$$

$$h_T=\frac{\dot{W}_T}{\dot{m}g}=Turbine\ Head$$

$$The\ Head\ Loss=h_{Loss}=\left(\frac{1}{g}\right)\left(u_2-u_1-\frac{\dot{Q}}{\dot{m}}\right)$$

$$\left(h_P+\frac{p_1}{\rho g}+z_1+\alpha_1\frac{\overline{V_1}^2}{2g}\right)_{Mech.part}=\left(h_T+\frac{p_2}{\rho g}+gz_2+\alpha_2\frac{\overline{V_2}^2}{2g}\right)_{Mech.part}+h_{Loss}$$

ENERGY PRINCIPLE

$$Pump\ Head = h_C = \frac{\dot{W}_C}{\dot{m}g}$$

$$\dot{W}_C = \dot{m}gh_C = \gamma Q h_C$$

$$\dot{W}_T = \dot{m}gh_T = \gamma Q h_T$$

$$Pump\ Efficiency = \eta_P = \frac{(\dot{W}_P)}{(\dot{W}_P)_{actual}}$$

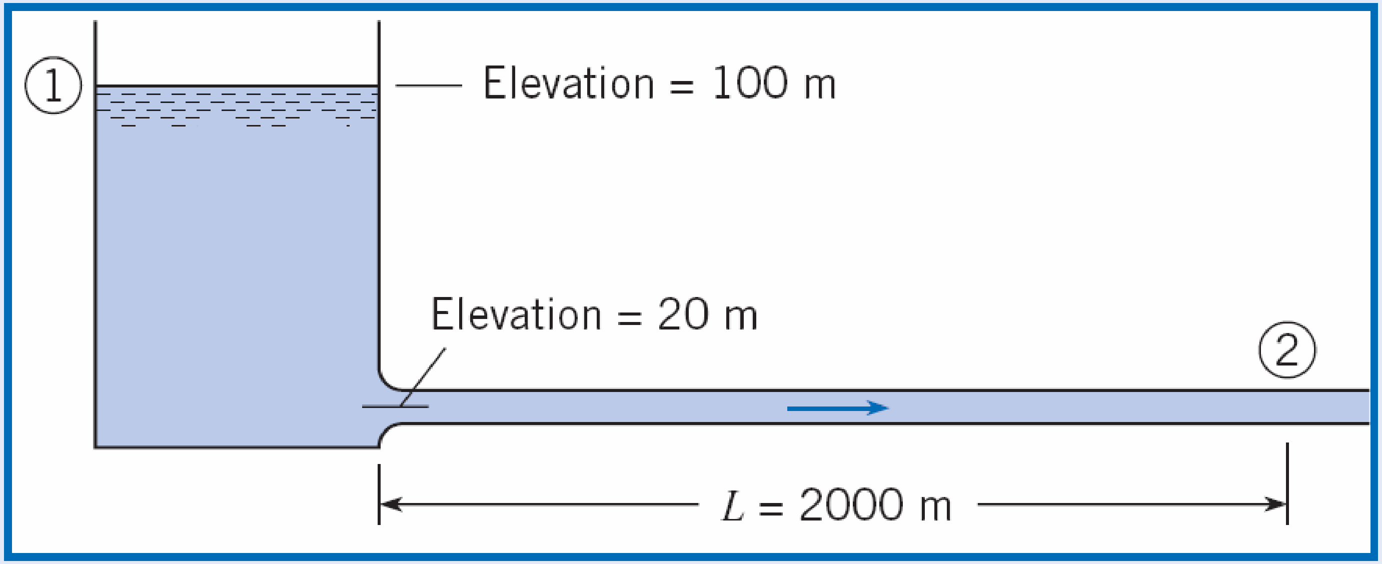
$$Turbine\ Efficiency = \eta_T = \frac{(\dot{W}_T)_{actual}}{(\dot{W}_T)}$$

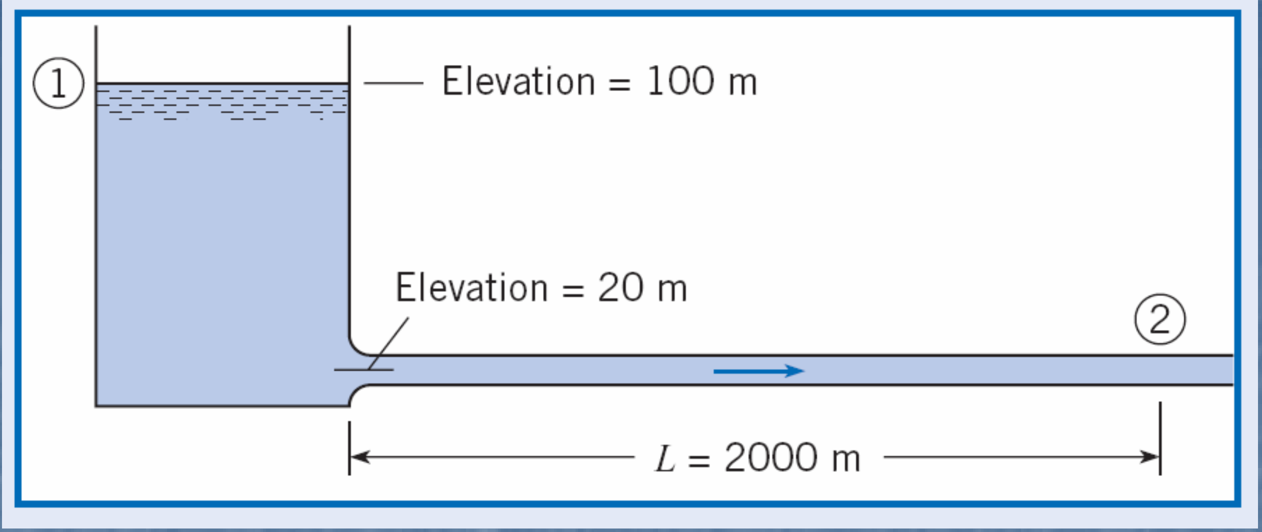
Example (7.3)

A horizontal pipe carries cooling water for a thermal power plant from a reservoir as shown. The head loss in the pipe is given as

$$\frac{0.02(L/D)V^2}{2g}$$

where L is the length of the pipe from the reservoir to the point in question, V is the mean velocity in the pipe, and D is the diameter of the pipe. If the pipe diameter is 20 cm and the rate of flow is 0.06 m^3/s, what is the pressure in the pipe at $L = 2000$ m? Assume $\alpha = 1$.





Given: $D = 20\,cm, \dot{Q} = 0.06\,m^3/s, \alpha = 1, h_{loss} = \frac{0.02(L/D)V^2}{2g}$

Find: $pressure\ at\ L = 2000\,m?$

Applying the energy equation between point 1 & 2

$$\left(h_P + \frac{p_1}{\rho g} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g}\right)_{Mech.part} = \left(h_T + \frac{p_2}{\rho g} + gz_2 + \alpha_2 \frac{\bar{V}_2^2}{2g}\right)_{Mech.part} + h_{Loss}$$

$$V_1 = 0, \quad p_1 = 0, z_1 = 100m, \quad z_2 = 20m, \quad V_2 = \frac{\dot{Q}}{A} = \frac{0.06}{\pi(0.02^2/4)} = 1.91 m/s, \quad \alpha_1 = \alpha_2 = 1$$

$$h_p = 0, \quad h_T = 0, \quad D = 20\,cm$$

$$h_{loss} = \frac{0.02(L/D)V^2}{2g} = \frac{0.02(2000/0{,}2^2)1.19^2}{2\times 9.81} = 37.2m$$

Substituting the values in the energy Eqn., the value of p_2 is found

$$\frac{p_2}{\gamma} = 100 - 20 - 0.186 - 0.0186L = 79.8 - 0.0186L$$

At $L = 2000$ m

$$\frac{p_2}{\gamma} = 79.8 - 37.2 = 42.6 \text{ m}$$

$$p_2 = 417.9 \text{ kPa}$$

END OF LECTURE (3)